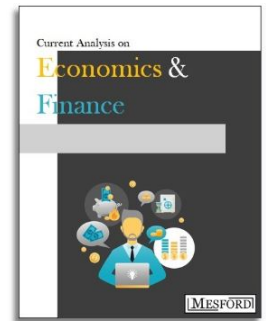# Growth, Corruption, and Business Cycles

Wei-Bin Zhang[*]

Ritsumeikan Asia Pacific University, Oita-ken, Japan.

Abstract:

The purpose of this paper is to deal with interdependence between bureaucratic corruption and economic development in a dynamic general equilibrium framework. The paper generalizes Zhang's growth model with corruption by allowing all constant parameters to be exogenously time-dependent (Zhang, 2017). The generalization makes the model robust as the original model can analyze effects of different exogenous perturbations on dynamics of the model. Zhang's model deals with an economy with one industrial and one public sector. The population is composed of workers and officials. Supply of public service is provided by officials and affects productivity of the industrial sector. The government is the sole financial supporter of the public sector. The government's income comes from taxes on the industrial sector. The industrial sector uses workers' labor inputs and capital. Officials may be corrupt. They may take bribes from the private sector and households. The officials accumulate wealth. Their corrupt money can either be saved or spent on consumption. How the corrupt money used also affects paths of economic growth. We simulate the generalized model to demonstrate business cycles due to exogenous periodic shocks. Comparative dynamic analysis provides some examples of business cycles.

1. INTRODUCTION

Corruption exists in all countries (Lui, 1996). Many theoretical models are proposed to reveal complexity of economic growth with corruption (Becker, 1968; Rose-Ackerman, 1999; Shi and Temzelides, 2004; and Agostino, et al. 2016). Different aspects of corruption are analyzed, even though there is no convergent conclusion on issues related to exact role of corruption on economic growth. Chea (2015: 187) states "From the theoretical background and empirical evidence from various studies spread in various countries in the world using different methods over the years, there are mostly negative findings of economic growth. Nevertheless, there are also positive findings found on the effects of corruption on economic growth." Dzhumashev (2014: 203) concludes the review on the literature of corruption as follows: "the existing literature lacks a more general approach for interpreting the role of governance, the size government, and the level of development in the relationship between corruption and growth." To properly model relations between corruption and growth, Zhang (2017)

proposes a two-sector two-group economy a model with corruption. The dynamic general equilibrium framework makes it possible to deal with different aspects of economic growth and corruption. The study makes a contribution to the literature by examining corrupt behavior within a dynamic equilibrium framework. The dynamic general equilibrium framework makes it possible to analyze different effects of corruption in an integrated manner. The main difference of this paper from Zhang's model is that this study makes all constant coefficients in Zhang's model to be time-dependent. This generalization enables us to examine effects of any exogenous time-dependent changes on the economic system.

There are many studies on business cycles due to various exogenous shocks or endogenous forces in the literature of theoretical economics (Zhang, 1991, 2005, 2006; Lorenz, 1993; Chiarella and Flaschel, 2000; Shone, 2002; Gandolfo, 2005; Puu, 2011). But there are a few theoretical economic models with interactions between endogenous wealth, corruption, income and wealth distribution between officials and workers

*Address correspondence to this author at the Ritsumeikan Asia Pacific University, Oita-ken, Japan.; Tel: 0977-78-1020; Fax: 0977-78-1123; E-mail: wbz1@apu.ac.jp

built with profound microeconomic foundation. This study shows existence of business cycles in a growth model with corruption and income and wealth distribution between officials and workers. The paper is organized as follows. Section 2 generalizes Zhang's growth model with corruption. Section 3 examines properties of the model and simulates model with time-independent parameters. Section 4 introduces different time-dependent shocks to demonstrate existence of business cycles. Section 5 concludes the study.

2. THE CORRUPTION-INFLUENCED GROWTH MODEL WITH EXOGENOUS SHOCKS

This section generalizes Zhang's model with wealth and corruption by introducing different sources of exogenous shocks. As this section is mainly concerned with making all the constant coefficients time-dependent parameters, we refer further explanations of the model to Zhang (2017). We consider an economy with industrial and public sectors. Like the single commodity in the Solow one-sector growth model, industrial good is used for investment and consumption. The price of the industrial good is unity. Capital depreciates at time-dependent rate, $\delta_k(t)$. We classify the population into officials and workers. Workers are engaged in the industrial sector and officials in the public sector. They are fully employed. Assets of the economy are owned by officials and workers. The industrial sector employs labor and capital as factor inputs. Markets are perfectly competitive. We use subscript index $j = 1$ and $j = 2$ to represent workers and officials respectively. Let $\bar{N}_j(t)$ stand for population of group j. We use $T_j(t)$ to represent work time of the representative household of group j. Total labor input of group j $N_j(t)$ is defined as follows:

$$N_j(t) = h_j(t)\bar{N}_j(t)T_j(t),$$

where $h_j(t)$ is an exogenous level of human capital of group j.

The Industrial Sector

Different from the traditional Cobb-Douglas functions, we apply the following form Cobb-Douglas function with time-dependent parameters

$$F(t) = A(t)G^{\theta(t)}(t)K_i^{\alpha_i(t)}(t)N_i^{\beta_i(t)}(t), \quad \alpha_i(t) + \beta_i(t) = 1, \quad A(t), \theta(t), \alpha_i(t), \beta_i(t) > 0, \quad (1)$$

where $A(t)$, $\theta(t)$, $\alpha_i(t)$ and $\beta_i(t)$ are parameters. Like Chen and Guo (2014), we assume that public good $G(t)$ affects productivity. We use $\varphi(t)$ to represent corruption rate on output level. Government decides tax rate on output $\tau(t)$. Here, we omit possible corruption on firms' capital and labor force inputs. Firms pat tax and corruption fee. Let $r(t)$ stand for rate of interest and $w_1(t)$ is worker's wage rate per unit of time. The marginal conditions are

$$r(t) + \delta_k(t) = \frac{\alpha_i(t)\bar{\varphi}(t)F(t)}{K_i(t)}, \quad w_1(t) = \frac{\beta_i(t)\bar{\varphi}(t)F(t)}{N_1(t)}, \quad (2)$$

where $\bar{\varphi}(t) \equiv 1 - \varphi(t) - \tau(t)$.

Workers' Disposable Income Budget Constraint

We use $\bar{k}_1(t)$ to express for the worker's wealth. In the absence of corruption on workers, the worker's current income is $r(t)\bar{k}(t) + w_1(t)T_1(t)$. The worker's disposable income is

$$\bar{k}_1(t) + r(t)\bar{k}_1(t) + h_1(t)w_1(t)T_1(t).$$

In this study consider that there are corruptions on workers. We use $\varphi_k(t)$ and $\varphi_w(t)$ to express for corruption rates on wealth and wage rate, respectively. Taking account of paying corruption fees to the official, we have the worker's disposable income $\hat{y}_1(t)$ as

$$\hat{y}_1(t) \equiv \bar{\varphi}_k(t)(1 + r(t))\bar{k}_1(t) + \bar{\varphi}_w(t)h_1(t)w_1(t)T_1(t), \quad (3)$$

where $\bar{\varphi}_k(t) \equiv 1 - \varphi_k(t)$ and $\bar{\varphi}_w(t) \equiv 1 - \varphi_w(t)$. The worker uses up the disposable income for saving $s_1(t)$ and consuming $c_1(t)$. The budget constraint is

$$c_1(t) + s_1(t) = \hat{y}_1(t). \quad (4)$$

We use $\bar{T}_j(t)$ to express for group j's leisure time and (fixed) available time for work and leisure by T_0. Time constraints are

$$T_j(t) + \bar{T}_j(t) = T_0, \quad j = 1, 2. \quad (5)$$

Substitute (3) and (5) into (4)

$$c_1(t) + s_1(t) + \bar{w}_1(t)\bar{T}_1(t) = \bar{y}_1(t) \equiv \bar{\varphi}_k(1 + r(t))\bar{k}_1(t) + \bar{w}_1(t)T_0, \quad (6)$$

in which $\bar{w}_1(t) \equiv \bar{\varphi}_w(t)h_1(t)w_1(t)$.

Officials' Disposable Income and Budget Constraint

We use $\bar{k}_2(t)$ to express for wealth owned by the representative official. It is assumed that the official is paid per unit of qualified work time in proportion to the worker's wage rate

$$\bar{w}_2(t) = u_0(t)h_2(t)w_1(t), \quad (7)$$

in which $u_0(t) > 0$ is given by the government budget. The official gets total corrupt income as follows:

$$w_c(t) \equiv \frac{\varphi(t)F(t) + \varphi_k(t)(1 + r(t))\bar{k}_1(t)\bar{N}_1(t) + \varphi_w(t)h_1(t)w_1(t)T_1(t)\bar{N}_1(t)}{\bar{N}_2(t)}. \quad (8)$$

Official's disposable income $\hat{y}_2(t)$ means

$$\hat{y}_2(t) \equiv (1 + r(t))\bar{k}_2(t) + \bar{w}_2(t)T_2(t) + w_c(t). \quad (9)$$

The official uses up disposable income for saving $s_2(t)$ and for consuming $c_2(t)$. The budget constraint implies:

$$c_2(t) + s_2(t) = \hat{y}_2(t). \quad (10)$$

Substitute (9) and (5) into (10)

$$c_2(t) + s_2(t) + \bar{w}_2(t)\bar{T}_2(t) = \bar{y}_2(t) \equiv (1 + r(t))\bar{k}_2(t)$$
$$+ \bar{w}_2(t)T_0 + w_c(t). \tag{11}$$

The Utility Functions and Optimal Decisions

Household j chooses three variables, $c_j(t)$, $s_j(t)$ and $\bar{T}_j(t)$ subject to the budget constraint. We use $U_j(t)$ to label for utility level which is dependent on the three variables as follows

$$U_j(t) = u_j(G(t), t)\bar{T}_j^{\sigma_{0j}(t)}(t)c_j^{\xi_{0j}(t)}(t)s_j^{\lambda_{0j}(t)}(t),$$
$$\sigma_{0j}(t), \xi_{0j}(t), \lambda_{0j}(t) > 0,$$

where $u_j(t)$ is a time-dependent variable, $\sigma_{0j}(t)$, $\xi_{0j}(t)$ and $\lambda_{0j}(t)$ are termed respectively the worker's/official's propensity to consume leisure, propensity to consume good, propensity to own wealth. The marginal conditions for maximizing U_j subject to (6)/(11) imply

$$\bar{w}_j(t)\bar{T}_j(t) = \sigma_j(t)\bar{y}_j(t), \quad c_j(t) = \xi_j(t)\bar{y}_j(t),$$
$$s_j(t) = \lambda_j(t)\bar{y}_j(t), \tag{12}$$

in which

$$\rho_j(t) \equiv \frac{1}{\sigma_{0j}(t) + \xi_{0j}(t) + \lambda_{0j}(t)},$$
$$\sigma_j(t) \equiv \rho_j(t)\sigma_{0j}(t), \quad \xi_j(t) \equiv \rho_j(t)\xi_{0j}(t), \quad \lambda_j(t) \equiv \rho_j(t)\lambda_{0j}(t).$$

Change in Wealth

Change in wealth is saving $s_j(t)$ minus dissaving $\bar{k}_j(t)$

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{13}$$

This equation simply means that change in the wealth is equal to saving minus dissaving.

The Public Sector

The government is sole financial supporter of the public sector. The public sector has income equal to $\tau(t)F(t)$. It is assumed that the sector uses officials' labor input $N_2(t)$, as sole input factor. The sector's production function is

$$G(t) = A_p(t)N_2^{\gamma(t)}(t), \quad A_p(t), \gamma(t) > 0. \tag{14}$$

The sector's budget is

$$\bar{w}_2(t)T_2(t)\bar{N}_2 = \tau(t)F(t). \tag{15}$$

For simplicity of analysis we neglect possible corruption on government.

Households Own Wealth

All national wealth is owned by the population

$$\bar{k}_1(t)\bar{N}_1(t) + \bar{k}_2(t)\bar{N}_2(t) = K(t). \tag{16}$$

Balance of Demand and Supply

Demand equals supply

$$\sum_{j=1}^{2} c_j(t)\bar{N}_j(t) + \sum_{j=1}^{2} s_j(t)\bar{N}_j(t)$$
$$+ \delta_k(t)K(t) = F(t) + K(t). \tag{17}$$

We generalized Zhang's model. The dynamic general equilibrium model shows interdependence between endogenous labor supply, economic structure, wealth growth, officials' corruption, income and wealth distribution, and public good supply subject to different exogenous shocks and different sources of taxation.

3. BEHAVIOR OF THE MODEL WITH CORRUPTION AND GROWTH

To express our result of analyzing the model properties, we define a variable

$$z(t) \equiv \frac{r(t) + \delta_k(t)}{w_1(t)}.$$

We show that the dynamics of model is given by two differential equations.

Lemma

The dynamics of the growth model with corruption is given by two differential equations with $z(t)$ and level of public good as variables

$$\dot{z}(t) = \tilde{f}_1(z(t), G(t), t),$$
$$\dot{G}(t) = \tilde{f}_2(z(t), G(t), t), \tag{18}$$

where $\tilde{f}_j(t)$ is are functions of $z(t)$, $G(t)$ and t defined in the appendix. Other variables are given as functions of $z(t)$, $G(t)$ and t by a computational procedure: $u_0(t)$ with (A.19) $\to \bar{k}_1(t)$ and $\bar{k}_2(t)$ from (A.20) $\to r(t)$ and $w_1(t)$ from (A.2) $\to \bar{w}_2(t)$ with (7) $\to w_c(t)$ by (A.6) $\to N_1(t)$ by (A.5) $\to N_2(t)$ with (A.13) $\to K(t)$ by (A.1) $\to \bar{y}_j(t)$ in (A.8) $\to F(t)$ from (A.3) $\to c_j(t)$, $s_j(t)$, and $\bar{T}_j(t)$ in (12) $\to N_2(t) = h_2(t)\bar{N}_2(t)T_2(t)$.

In order to see how periodic shocks affect paths of the system, we first fix all parameters. As in Zhang (2017). parameter values are taken as follows

$$\bar{N}_1 = 100, \quad \bar{N}_2 = 10, \quad T_0 = 24, \quad h_1 = 2.5,$$
$$h_2 = 2.5, \quad \alpha_i = 0.3, \quad A = 1.2, \quad A_p = 0.9,$$
$$\xi_{01} = 0.15, \quad \lambda_{01} = 0.4, \quad \sigma_{01} = 0.3, \quad \xi_{02} = 0.15,$$
$$\lambda_{02} = 0.5, \quad \sigma_{02} = 0.15, \quad \theta = 0.1, \quad \gamma = 0.3,$$

$$\tau = 0.01, \quad \varphi = 0.03, \quad \varphi_k = 0.05, \quad \varphi_w = 0.05, \quad \delta_k = 0.05. \tag{19}$$

Initial conditions are taken on the values

$$z(0) = 0.21, \quad G(0) = 2.4.$$

We depict the changes of the variables in Fig. (**1**).
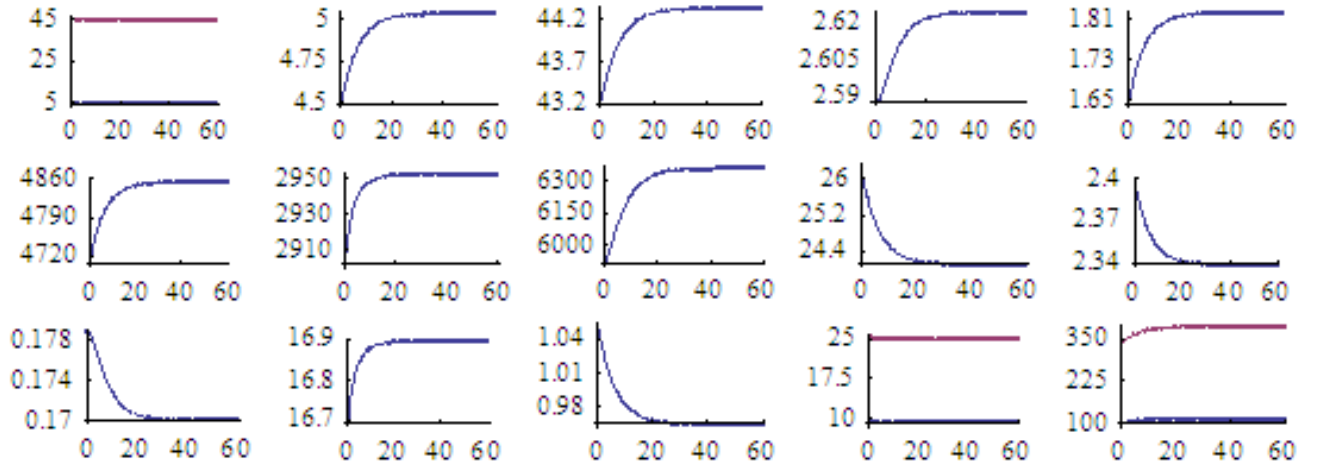
We calculate the equilibrium values of the variables
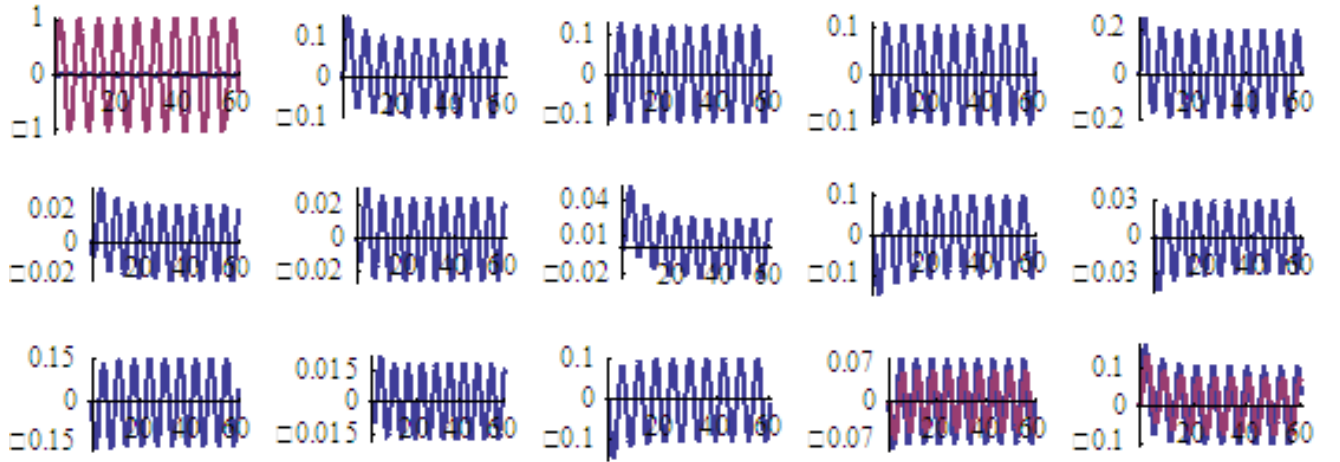
Fig. (1). The Motion of the Dynamic System.

Fig. (2). The Corruption Rate on the Output Level is Periodically Changed.

$u_0 = 1.82$, $w_c = 45.4$, $w_1 = 1.12$, $\bar{w}_2 = 5$,
$\bar{W}_1 = 44.3$, $\bar{W}_2 = 4.86$, $N_1 = 2953.3$,
$N_2 = 24.1$, $\bar{K}_1 = 2485.4$, $\bar{K}_2 = 3868.8$,
$F = 4855$, $G = 2.34$, $\bar{k}_1 = 24.9$,
$\bar{k}_2 = 386.9$, $T_1 = 16.9$, $\bar{T}_1 = 7.1$, $T_2 = 1$,   (20)
$\bar{T}_2 = 23$, $c_1 = 9.3$, $c_2 = 116$,

where $\bar{W}_j = \bar{w}_j T_j$. The eigenvalues are

-0.425, -0.16.

The negative eigenvalues implies local stability. This is important as it validity of comparative dynamic analysis.

4. COMPARATIVE DYNAMIC ANALYSIS

The previous section showed how to apply a computational procedure to depict the movement of the economic system and showed a case when all the parameters are constant. The case is also examined in Zhang (2017). This section shows how the system changes over time when it is subject to different periodic time-dependent shocks. We apply symbol $\bar{\Delta}x(t)$ to express for the change rate of variable, $x(t)$, in percentage due to changes in parameter value.

4.1. The Corruption Rate on Output Level is Periodically Changed

We first study how the economic system is affected when the corruption rate on output level is changed periodically as follows:

$\varphi(t) = 0.03 + 0.001\sin(t)$.

The simulation result is plotted in Fig. (**2**). The variables oscillate around the corresponding variables in Fig. (**2**).

4.2. The Corruption Rate on Capital Income is Periodically Perturbed

We now study a case that the corruption rate on capital income is periodically perturbed as follows:

$\varphi_k(t) = 0.05 + 0.01\sin(t)$.

The result is illustrated in Fig. (**3**). The system periodically oscillates around the paths depicted in Fig. (**1**).

4.3. The Corruption Rate on Wage Income is Periodically Perturbed

We now study a case that the corruption rate on wage income is periodically perturbed as follows:
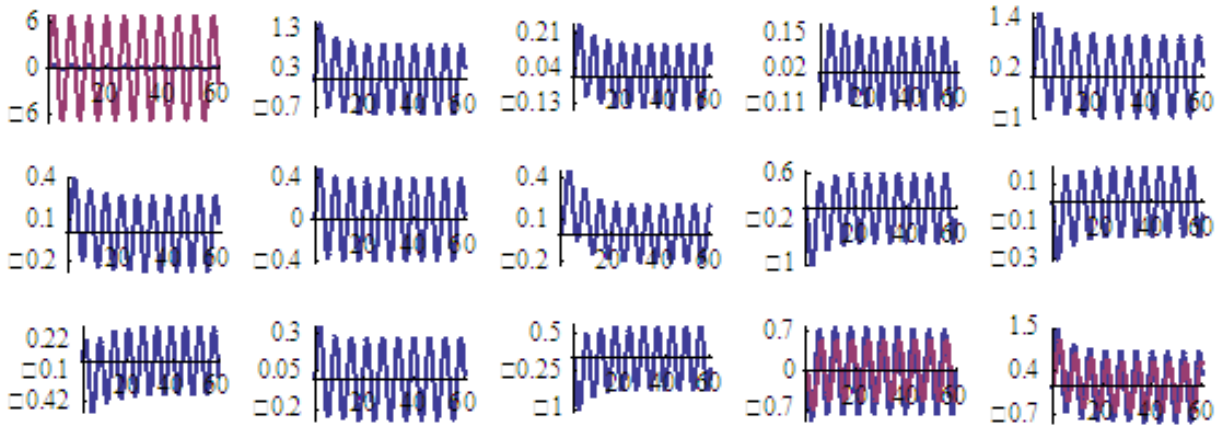
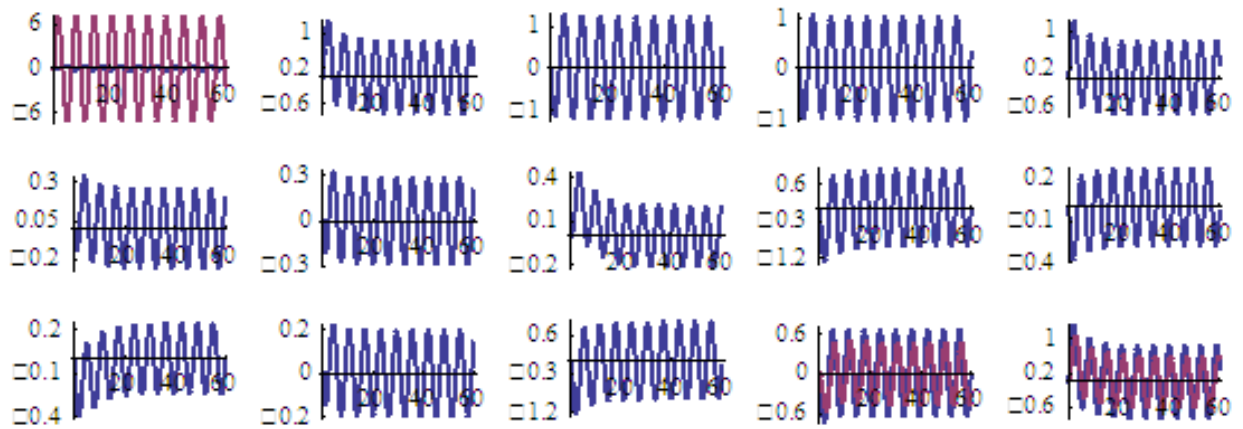**Fig. (3).** The Corruption Rate on Capital Income is Periodically Perturbed.

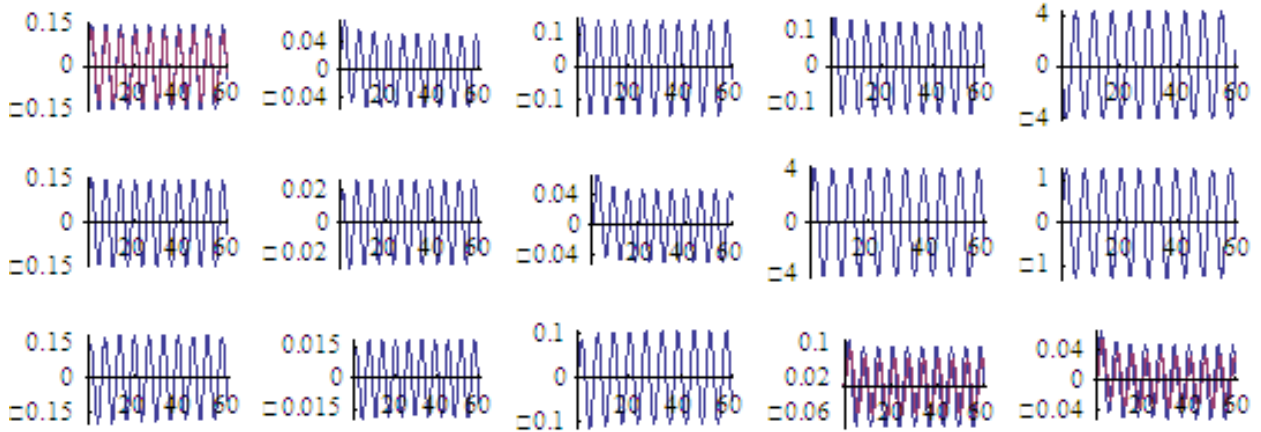**Fig. (4).** The Corruption Rate on Wage Income is Periodically Perturbed.

Fig. (5). Officials' Human Capital Periodically Oscillates.

$$\varphi_w(t) = 0.05 + 0.01\sin(t).$$

The result is illustrated in Fig. (**4**). The system periodically oscillates around the paths depicted in Fig. (**1**).

4.4. Officials' Human Capital Periodically Oscillates

We now allow officials' human capital to periodically oscillate as follows:

$$h_2(t) = 2.5 + 0.1\sin(t).$$

The simulation result is plotted in Fig. (**5**).

5. CONCLUDING REMARKS

This study generalized Zhang's growth model with corruption by allowing all constant parameters to be exogenously time-dependent. The generalization makes the model robust as the general model can analyze effects of different exogenous perturbations on dynamics of the model. Zhang's model deals with an economy with one industrial and one public sector.

The population is composed of workers and officials. Supply of public service is provided by officials and affects productivity of the industrial sector. The government is the sole financial supporter of the public sector. The government's income comes from taxes on the industrial sector. The industrial sector uses workers' labor inputs and capital. Officials are corrupt. They take bribes from the private sector and households. We simulated the model to demonstrate business cycles due to exogenous periodic shocks. The comparative dynamic analysis provided some examples of exogenous business cycles. As mentioned before, there is a large number of literature on corruption. There are relatively few macroeconomic growth models with endogenous business cycles in a general equilibrium framework. Our model can be extended in various ways. For instance, channels of corruption are far more complicated than the ones described in this study. We don't have dynamic interdependence between institutional structure and corruption networks. It is important to allow heterogeneous workers and heterogeneous officials.

APPENDIX: CHECKING THE LEMMA

From (2) we get

$$z \equiv \frac{r + \delta_k}{w_1} = \frac{N_1}{\bar{\beta}_i K}, \qquad (A.1)$$

where $\bar{\beta}_i \equiv \beta_i / \alpha_i$. From (1), (2), and (A.1), we get

$$r(z, G) = A_r \bar{\varphi} - \delta_k, \quad w_1(z, G) = A_w \bar{\varphi}, \qquad (A.2)$$

In which

$$A_r(z, G) = \alpha_i A \bar{\beta}_i^{\beta_i} G^\theta z^{-\alpha_i}, \quad A_w(z, G) = \frac{\beta_i A G^\theta z^{-\alpha_i}}{\bar{\beta}_i^{\alpha_i}}. \text{With} \qquad (1)$$

and (2) we have

$$F = N_1 f, \qquad (A.3)$$

in which $f \equiv A G^\theta / \bar{\beta}_i^{\alpha_i} z^{\alpha_i}$. From (12) we solve

$$\bar{T}_1 = \sigma_1 T_0 + \frac{\sigma_1 \bar{\varphi}_k (1 + r) \bar{k}_1}{\bar{w}_1}. \qquad (A.4)$$

From (A.4), (5), and the definition of N_1, we have

$$N_1 = n_0 - n \bar{k}_1, \qquad (A.5)$$

in which

$$n_0 \equiv (1 - \sigma_1) h_1 \bar{N}_1 T_0, \quad n \equiv \frac{\sigma_1 h_1 \bar{N}_1 \bar{\varphi}_k (1 + r)}{\bar{w}_1}.$$

From (8) and (A.3) we have

$$w_c = q_z N_1 + q_k \bar{k}_1 + q_0, \qquad (A.6)$$

in which we also use (12) and (6) and

$$q_z \equiv \frac{\varphi f}{\bar{N}_2}, \quad \varphi_0 \equiv \frac{\varphi_w \sigma_1 \bar{N}_1}{\bar{\varphi}_w}, \quad q_k \equiv \frac{(\varphi_k \bar{N}_1 - \varphi_0 \bar{\varphi}_k)(1 + r)}{\bar{N}_2},$$

$$q_0 \equiv \frac{\varphi_w h_1 w_1 T_0 \bar{N}_1 - \varphi_0 \bar{w}_1 T_0}{\bar{N}_2}.$$

With (18), (17) and (A.1) we solve

$$\bar{k}_1 \bar{N}_1 + \bar{k}_2 \bar{N}_2 = \frac{N_1}{z \bar{\beta}_i}. \qquad (A.7)$$

Substitute (A.5) into (A.7)

$$\bar{k}_1 = \frac{\tilde{n}_0 n_0}{z \bar{\beta}_i} - \bar{k}_2 \tilde{n}_0 \bar{N}_2, \qquad (A.8)$$

in which

$$\tilde{n}_0 = \left(\bar{N}_1 + \frac{n}{z \bar{\beta}_i} \right)^{-1}.$$

With (A.6), (6), and (11) we solve

$$\bar{y}_1 = \bar{\varphi}_k (1 + r) \bar{k}_1 + \bar{\varphi}_w w_1 T_0,$$

$$\bar{y}_2 = (1 + r) \bar{k}_2 + \bar{w}_2 T_0 + q_z N_1 + q_k \bar{k}_1 + q_0. \quad (A.9)$$

With the time constraint and (12) we get

$$T_2 = T_0 - \frac{\sigma_2 \bar{y}_2}{\bar{w}_2}. \quad (A.10)$$

Substitute (A.9) into (A.10

$$T_2 = (1 - \sigma_2) T_0 - \sigma_2 \left(\frac{(1 + r) \bar{k}_2 + q_z N_1 + q_k \bar{k}_1 + q_0}{\bar{w}_2} \right). \quad (A.11)$$

Substitute (A.5) and (A.11) into $N_2 = h_2 \bar{N}_2 T_2$

$$N_2 = \tilde{n} - \bar{n} \bar{k}_1 - \frac{(1 + r) h_2 \bar{N}_2 \sigma_2 \bar{k}_2}{\bar{w}_2}, \quad (A.12)$$

in which

$$\tilde{n} = \sigma_2 h_2 \bar{N}_2 \left(\left(\frac{1}{\sigma_2} - 1 \right) T_0 - \frac{q_z n_0 + q_0}{\bar{w}_2} \right),$$

$$\bar{n} = \sigma_2 h_2 \bar{N}_2 \left(\frac{q_k - n q_z}{\bar{w}_2} \right).$$

From (A.8) and (A.12) we have

$$N_2 = \hat{n}_0 + \hat{n} \bar{k}_2, \qquad (A.13)$$

in which

$$\hat{n}_0 \equiv \tilde{n} - \frac{\tilde{n}_0 \bar{n} n_0}{z \bar{\beta}_i}, \quad \hat{n} \equiv \bar{n} \tilde{n}_0 \bar{N}_2 - \frac{(1 + r) h_2 \bar{N}_2 \sigma_2}{\bar{w}_2}.$$

Equations (14) and (A.13) imply

$$\bar{k}_2 = f_2(z, G, u_0) \equiv \left(\frac{G}{A_p} \right)^{1/\gamma} \frac{1}{\hat{n}} - \frac{\hat{n}_0}{\hat{n}}. \qquad (A.14)$$

Substitute (A.14) into (A.8)

$$\bar{k}_1 = f_1(z, G, u_0) \equiv \frac{\tilde{n}_0 n_0}{z \bar{\beta}_i} - f_2 \tilde{n}_0 \bar{N}_2. \qquad (A.15)$$

With (15) and (A.3)

$$\left(\frac{1}{\sigma_2} - 1\right)T_0\,\bar{w}_2 - (1 + r)\bar{k}_2 - \hat{q}_k\,\bar{k}_1 = f_0, \qquad (A.16)$$

in which we apply (A.11) and

$$f_0 \equiv q_0 + \left(\frac{\tau f}{\sigma_2 \bar{N}_2} + q_z\right)n_0, \quad \hat{q}_k \equiv q_k - \left(\frac{\tau f}{\sigma_2 \bar{N}_2} + q_z\right)n.$$

Substitute (A.8) into (A.16)

$$\bar{w}_2 - \tilde{f}_0\,\bar{k}_2 = \hat{f}_0, \qquad (A.17)$$

in which

$$\tilde{f}_0(z, G, u_0) \equiv \big((1 + r) - \hat{q}_k\,\tilde{n}_0\,\bar{N}_2\big)t_2,$$

$$\hat{f}_0(z, G, u_0) \equiv \left(\frac{\hat{q}_k\,\tilde{n}_0\,n_0}{z\,\bar{\beta}_i} + f_0\right)t_2, \quad t_2 \equiv \left(\frac{1}{\sigma_2} - 1\right)^{-1}\frac{1}{T_0}$$

From (A.14) and (A.17) we get

$$m_0\,\bar{w}_2 + \frac{\tilde{f}_0\,\tilde{n}\bar{w}_2}{\sigma_2\,h_2\,\bar{N}_2} = m_1, \qquad (A.18)$$

where we apply the definitions of $\hat{n}$ and

$$m_0(z, G) \equiv (q_k - nq_z)\tilde{n}_0\,\bar{N}_2 - (1 + r) - \frac{\tilde{f}_0}{\sigma_2\,h_2\,\bar{N}_2}\left(\frac{G}{A_p}\right)^{1/\gamma},$$

$$m_1(z, G) \equiv \left(\frac{\tilde{f}_0\,n_0}{z\,\bar{\beta}_i} + \bar{N}_2\hat{f}_0\right)(q_k - nq_z)\tilde{n}_0 - (1 + r)\hat{f}_0.$$

Insert the definition of $\tilde{n}$ in (A.18)

$$u_0(z, G) = \big((q_z n_0 + q_0)\tilde{f}_0 + m_1\big)\left(m_0 + \frac{\tilde{f}_0}{t_2}\right)^{-1}\frac{1}{h_2\,w_1}, \qquad (A.19)$$

in which we use (7). Substitute (A.19) in (A.14) and (A.15)

$$\bar{k}_j = f_j(z, G, t). \quad (A.20)$$

We showed the computational procedure in the lemma. Take derivatives of (A.20) with respect to t

$$\dot{\bar{k}}_j = \frac{\partial f_j}{\partial z}\dot{z} + \frac{\partial f_j}{\partial G}\dot{G} + \frac{\partial f_j}{\partial t}, \quad j = 1, 2. \qquad (A.21)$$

From the above procedure and (13), we have

$$\dot{\bar{k}}_j = \bar{f}_j(z, G, t) \equiv s_j - \bar{k}_j. \qquad (A.22)$$

Insert (A.22) in (A.21)

$$\frac{\partial f_j}{\partial z}\dot{z} + \frac{\partial f_j}{\partial G}\dot{G} = \bar{f}_j - \frac{\partial f_j}{\partial t}, \quad j = 1, 2. \qquad (A.23)$$

Solve (A.23)

$$\dot{z} = \tilde{f}_1(z, G, t), \\ \dot{G} = \tilde{f}_2(z, G, t). \qquad (A.24)$$

We confirmed the lemma.

CONFLICT OF INTEREST

The author declares that there are no conflicts of interest.

REFERENCES

Becker, G.: Crime and punishment: an economic approach. J Polit Econ 76: 169-217 (1968).

Chea, C.C.: Empirical studies: corruption and economic growth. Am J Econ 5: 183-188 (2015).

Chiarella, C., Flaschel, P.: The dynamics of keynesian monetary growth: macro foundations. Cambridge University Press, Cambridge (2000).

D'Agostino, G., Dunne, P., Pieroni, L.: Government spending, corruption and economic growth. World Dev 84: 190-205 (2016).

Dzhumashev, R. Corruption and growth: the role of governance, public spending, and economic development. Econ Model 37: 201-215 (2014).

Gandolfo, G.: Economic dynamics. Springer, Berlin (2005).

Lorenz, H.W.: Nonlinear dynamic economics and chaotic motion. Springer-Verlag, Berlin (1993).

Lui, F.T.: Three aspects of corruption. Contemp Econ Policy 14: 26-29 (1996).

Puu, T.: Nonlinear economic dynamics. Springer, Berlin (2011).

Rose-Ackerman, S.: Corruption and government: causes, consequences, and reform. Cambridge University Press, Cambridge (1999).

Shi, S.Y., Temzelides, T.: A model of bureaucracy and corruption. Int Econ Rev 45: 873-906 (2004).

Zhang, W.B.: Synergetic economics. Springer-Verlag, Heidelberg (1991).

Zhang, W.B.: Differential equations, bifurcations, and chaos in economics. World Scientific, Singapore (2005).

Zhang, W.B.: Discrete dynamical systems, bifurcations and chaos in economics. Elsevier, Amsterdam (2006).

Zhang, W.B.: Corruption and growth in a dynamic general equilibrium model. Eur J Multidiscip Stud 8: 256-73 (2017).